Tip-Tilt-Focus Estimation at PAA for GEO Feeder uplinks aided by Laser Guide Star

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Introduction

Context: High data rate optical ground to GEO links impacted by anisoplanatism

Main issue: atmospheric turbulence
Random fluctuation of coupling efficiency.

Solution: Adaptive Optics (AO)

Uplink correction: not optimal
Because of link geometry: Point ahead angle (PAA).
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How to improve information signal reliability over this fading channel?

→ Improve the coupled flux statistics by optimizing the pre-compensation phase
State of the art
How to optimize the pre-compensation phase at PAA?

Technique 1: Classical pre-compensation [Tyson-1996]
Shared phase correction with the downlink
→ Full anisoplanatism
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Provides $\Phi$ measurements at PAA
→ Tip-Tilt Focus indetermination
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**Technique 3: Phase estimation at PAA** [Lognoné-2023]
Based on on-axis \( \Phi \) and \( \chi \) measurements

\( \Psi(\alpha = \alpha_{\text{PAA}}) \) \hspace{2cm} \( \Psi(\alpha = 0) \)

\( \alpha_{\text{PAA}} \)
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Based on on-axis $\Phi$ and $\chi$ measurements

We propose:
To combine 2 and 3 to estimate the uplink tip tilt and focus at PAA by incorporating high order LGS measurements in the phase estimation.
System model

System:
→ Reciprocal approach to compute the phase error and the reciprocal coupled flux
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Phase analysis:
- Zernike Modal formalism:
  \[ \Phi = (a_2, ..., a_N) \text{ and } \chi = (b_1, ..., b_N) \]
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LGS system assumptions:
- Punctual monostatic LGS
  - Perfect high order measurements
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LGS system assumptions:
- Punctual monostatic LGS
  ➢ Perfect high order measurements

Adaptive optics assumptions:
Error budget: Anisoplanatism and fitting
Benchmark

Systems without LGS:
- Classical case, full anisoplanatism:
  \[ \Phi_{\text{res, classic}} = \Phi_{\text{PAA}} - \Phi_0 \]
- MMSE:
  \[ \Phi_{\text{res, MMSE}} = \Phi_{\text{PAA}} - \Phi_{\text{MMSE}} \]
Benchmark

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Systems with LGS:
We suppose perfect measurements on high order modes:

\[ \Phi_{\text{res, LGS}} = \left( \begin{array}{c} \Phi_{\text{PAA, TTF}} \\ \Phi_{\text{PAA}} \end{array} \right) - \left( \begin{array}{c} \Phi_{\text{corr, TTF}} \\ \Phi_{\text{PAA}} \end{array} \right) \]
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As we study LGS systems:
→ Focus on Tip Tilt and Focus analysis
Tip tilt and focus estimation
Theoretical phase estimator

Phase error:

\[ e = \Phi_{\text{res}} = \Phi_{\text{PAA}} - \hat{\Phi}_{\text{PAA}} \]

Linear estimator:

\[ \hat{\Phi}_{\text{PAA}} = R y_m \]

MMSE estimation:

\[
R_{\text{MMSE}} = \min_{R} \mathbb{E} \left[ (\Phi_{\text{PAA}} - R y_m)^T (\Phi_{\text{PAA}} - R y_m) \right]
= \Gamma_{\Phi y_m(\alpha)} \Gamma_{y_m y_m(0)}^{-1}
\]
Tip tilt and focus estimation
Specification of the measurement vector

Reminder: \( \mathbf{R}_{MMSE} = \Gamma_{\Phi} \gamma_m(\alpha) \Gamma_{\gamma_m\gamma_m}(0)^{-1} \)
Tip tilt and focus estimation
Specification of the measurement vector

Reminder: \( R_{MMSE} = \Gamma_{\Phi y_m}(\alpha)\Gamma_{y_m y_m}(0)^{-1} \)

New method
\( \rightarrow \) Estimate of \( \Phi_{PAA,TTF} \)

\( y_{m,LGS} = \begin{pmatrix} \Phi_0 \\ \chi_0 \\ \Phi_{PAA,HO} \end{pmatrix} \)
Tip tilt and focus estimation

Specification of the measurement vector

Reminder: $R_{MMSE} = \Gamma \Phi y_m(\alpha) \Gamma y_m y_m(0)^{-1}$

New method

$\to$ Estimate of $\Phi_{PAA,TTF}$

$$y_{m,LGS} = \begin{pmatrix} \Phi_0 \\ \chi_0 \\ \Phi_{PAA,HO} \end{pmatrix}$$

where $\Phi_0$, $\Phi_{PAA,HO}$ and $\chi_0$ are vectors of the projections of the physical quantities onto the Zernike polynomial basis, as:

$$\Phi_0 = (a_2^0 ... a_n^0)^T, \Phi_{PAA,HO} = (a_5^{\alpha_{PAA}} ... a_n^{\alpha_{PAA}})^T,$$

and $\chi_0 = (b_1^0 ... b_n^0)^T$
Tip tilt and focus estimation
Specification of the measurement vector

Reminder: \( R_{MMSE} = \Gamma \Phi y_m(\alpha) \Gamma y_m(0)^{-1} \)

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where \( \Phi_0, \Phi_{PAA,HO} \) and \( \chi_0 \) are vectors of the projections of the physical quantities onto the Zernike polynomial basis, as:

\[
\Phi_0 = (a_2^0 ... a_n^0)^T, \quad \Phi_{PAA,HO} = (a_5^{\alpha_{PAA}} ... a_n^{\alpha_{PAA}})^T
\]

and \( \chi_0 = (b_1^0 ... b_n^0)^T \)

Analytical estimator depends on:
- \( C_n^2 \) profile
- OGS parameters: \( D, k_0, \alpha_{PAA} \)
Estimator performances
OGS and atmospheric parameters

\[ r_0 \text{ at } 1550 \text{ nm} \] 4,0 cm
\[ \sigma^2 \chi \] 0,08
\[ \theta_0 \] 6,8 µrad
\[ (v_g, v_t) \] (10, 30) m.s\(^{-1}\)

\[ C_2 \text{ m}^{-2/3} \] vs. Height (km)

OGS parameters
- D 60 cm
- \( \theta_{\text{elevation}} \) 30°
- \( \theta_{\text{PAA}} \) 18,5 µrad
- \( \lambda \) 1550 nm

AO parameters
- \( N_{\text{AO}} \) 136
- \( f_{\text{samp}} \) 4,7 kHz

\[ \rightarrow 47000 \text{ E2E samples generated} \]
Estimator performances
Gain on Tip Tilt and Focus residual phase variance

\[ \sigma^2(\Phi_{\text{res, } i}) \text{ (rad}^2) \]

- **Classic**
- **MMSE_{\Phi_x}**

**I. Introduction**
**II. System and benchmark**
**III. Tip tilt and focus estimation**
**IV. Estimator performance**
**IV. Discussion**
Estimator performances
Gain on Tip Tilt and Focus residual phase variance

We observe:
→ Tip reduced by 70%, tilt by 50%, focus by 80% with respect to the classical case.

\[
\begin{align*}
\sigma_{TTF,\text{classic}}^2 &= 0.53 \text{ rad}^2 \\
\sigma_{TTF,\text{MMSE}_{\Phi \chi}}^2 &= 0.29 \text{ rad}^2 \\
\sigma_{TTF,\text{MMSE}_{\Phi \chi,LGS}}^2 &= 0.19 \text{ rad}^2
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Estimator performances
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Adding LGS high order measurements brings information and therefore improves the estimation.

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Estimator performances
Coupled flux statistics

Gain with respect to the classical case at $P(f \leq F_t) = 10^{-3}$

- **MMSE$\Phi_X$**: 13 dB
- **LGS TTF Classic**
- **LGS TTF MMSE$\Phi_X, LGS$**

![Graph showing coupled flux threshold $F_t$ vs. probability $P(f \leq F_t)$]
Estimator performances
Coupled flux statistics

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![Graph showing coupled flux threshold $F_t$ (dB)](image)

- Classic (No LGS)
- $MMSE_{\phi_\chi}$ (No LGS)
- LGS, TTF classic
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Estimator performances
Coupled flux statistics

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![Graph showing coupled flux statistics for different methods]
# Estimator performances

## Coupled flux statistics

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1) $\text{MMSE}_{\Phi \chi}$ performs better than LGS with Tip tilt focus classic

2) **For LGS systems**: better performance using LGS phase high orders in the estimation
Estimator performances

Coupled flux statistics

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1) \( \text{MMSE}_{\phi_X} \) performs better than LGS with Tip tilt focus classic

2) For LGS systems: better performance using LGS phase high orders in the estimation

This estimation method reduces the occurrence of deep fades: \( \Rightarrow \) Relax constraints on the link budget.
Estimator performances
Temporal statistics

![Graph showing coupled flux over time with different estimators]
Estimator performances
Temporal statistics

We observe:
- Fading depth and duration reduced with respect to every other methods
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⇒ Reduce the link latency and satellite system complexity.
Conclusion and perspectives

- We derived a new **analytical estimator** relying on **on-axis phase and log-amplitude** and **laser guide star high order measurements**.

- Laser guide star high order measurements bring information to the estimation and therefore further **decrease the tip tilt and focus residual phase variance**.

- This has the consequence to highly improve the statistics and temporal characteristics of the coupled flux aboard the satellite:
  - Relax **link budget constraints**
  - Relax constraints on **interleavers duration** : decreased latency and system complexity aboard the satellite.

**Perspectives**

- **Idealized LGS**: this is the upper limit of the performance we can get
  - Need to study a more realistic LGS system (width of the source, noise, lack of stabilization)

- **Toward implementation**:
  - Cn2 profile reconstruction to ensure the estimator robustness.
FEELINGS Ground Station

- $D = 60$ cm robotised telescope (20 cm subaperture option for uplink)
- Compatible with TELEO payload
- GEO and LEO tracking ability
- Compatible with phase and amplitude modulation
- AO (FELIN): 17x17 SH, 4.7kHz, Alpao DM292 + fast TT mirror
- ONERA’s RTC (developed by Shakti)
- OS: Laboratory based (LV + IDL), scripts, not automated
- In-house high power amplifier
- Weather station, Integral Sky Monitor, Miratlas
- In-house automated post-processing of the experimental data
Thank you for your attention!


